

Mathematics and Statistics Awareness Month Problems

College Problems

C1. The sum of five real numbers is 7; the sum of their squares is 10. Find the minimum and maximum possible values of any one of the numbers.

C2. Prove that $x^4 - x + 1$ is greater than zero for all real numbers x

C3. Find all positive integers a, b and c for which $3^a + 6^b + 8^c = 244$.

C4. Solve the system of non-linear equations: $ab + c = 6$; $bc + a = 6$; $ca + b = 6$.

C5. Find all powers of 2 such that, after deleting the first digit, another power of 2 remains. (For example, $2^5 = 32$. On deleting the initial 3, we are left with $2 = 2^1$.) Numbers are written in standard decimal notation, with no leading zeroes.

C6. Alice: Have you been hitting the bottle again? Yesterday this bottle of whiskey was full, but today there are only 14 centimeters of whiskey in it.

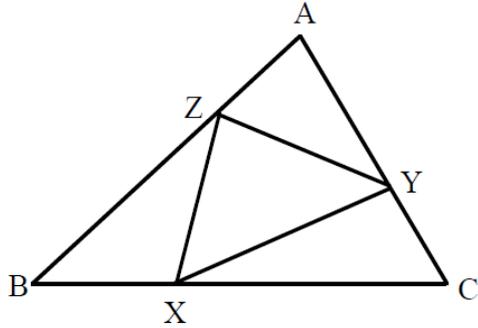
Bob (turning the bottle upside down): Fourteen? Look: There are still 19 cm of whiskey in the bottle.

Given that a full bottle holds 750 cubic centimeters (cc), the bottle is 27 centimeters from top to bottom, and forms a cylinder of diameter 7 centimeters. (You may assume that the bottom of the bottle is flat and the stem at the top is stoppered flush with the top of the cylindrical part of the bottle.) How much whiskey is in the bottle, in cc?

C7. A table with 5 rows and 5 columns is filled with nonnegative integers. With an add move we can select one of the 5 rows or one of the 5 columns and add 1 to each number in that row or column. With a subtract move we can select one of the 5 rows or one of the 5 columns and subtract 1 from each number in that row or column. Suppose that we can find a sequence of add moves and subtract moves that will reduce our table of nonnegative integers to a table filled with zeros only. Prove that in that case the original table of nonnegative integers can also be reduced to all zeros using only subtract moves.

C8. Each member of a sports team is a friend of at least six others on the team, and yet there are no four members of the team such that each of them is a friend of the other three. Find (with proof, of course) the smallest size team for which this is possible. (You should assume that the “friend” relationship is symmetric. In other words, if A is a friend of B , then also, B is a friend of A .)

C9. In triangle ABC , point X lies on side BC , one third of the way from B to C . Similarly, Y is chosen on CA , one third of the way from C to A , and Z lies on AB , one third of the way from A to B . If the area of triangle ABC is 1 unit, find the area of triangle XYZ .



C10. Let S be the circle of radius 1 centered at $(1, 0)$. Consider a circle C centered at the origin and let A and B mark the intersection of C 's upper half with the positive y-axis and S . Extend the line AB rightward, letting X be its x-intercept. What happens to X as C becomes smaller and smaller?

