

Mathematics and Statistics Awareness Month 2018

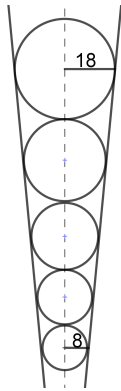
High School Level Problems

B1. Find all non-negative integer pairs (x, y) which satisfy the equation $x^2 - x + y = 100$.

B2. An empty rectangular swimming pool is 3 feet deep and its bottom measures 8 feet by 12 feet (the sides are vertical so that the top of the pool is also an 8 by 12 rectangle). A lizard, located at one of the top corners of the pool, must walk along the surface (sides and bottom) of the pool to reach the diagonally opposite top corner. What is the length of the shortest such path?

Note: The lizard can walk on any of the five surfaces of the pool and it can walk along any of the edges of the pool.

B3. Five marbles of various sizes are placed in a conical funnel. Each marble is in contact with the adjacent marble(s). Also, each marble is in contact all around the funnel wall. The smallest marble has a radius of 8mm. The largest marble has a radius of 18mm. A cross-sectional view is provided below. What is the radius of the middle marble?



B4. The standard denomination US coins are: penny (1 cent), nickel (5 cents), dime (10 cents), quarter (25 cents), half-dollar (50 cents), AND dollar (100 cents). Kennedy Half-Dollars and Native American Dollars are circulating coins produced as collectibles, not for everyday transactions. However, they may be still used as legal tender.

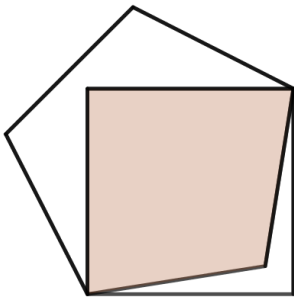
Suppose that you are given $n > 0$ coins in each of the denominations. (n pennies, n nickels, *etc.*) What is the smallest value of n for which it is impossible to select n of the $6n$ coins you have so that the combined value of these n coins is exactly a dollar?

B5. How many digits does the number $2^{1000} \cdot 3 \cdot 5^{996} \cdot 4$ have?

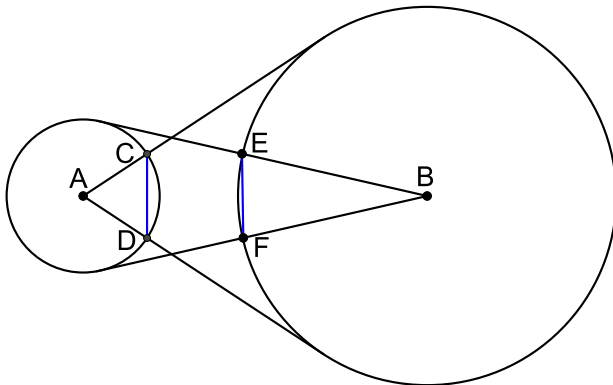
B6. A box contains an assortment of colored balls, with equal numbers of each color. When 20 balls of a new color are added to the box, the probability of drawing (at random without replacement) two balls of the same color does not change. How many balls are in the box initially, *i.e.*, how many balls are in the box before the extra balls are added?

B7. Show that $\frac{4n+3}{6n+4}$ is irreducible for every natural number n .

B8. The square in the figure below has sides of length 1. The pentagon, which shares 2 vertices with the square, is a regular pentagon. Find the area of the part of the interior of the pentagon that is also inside the square. This region is shaded in the figure below.



B9. The figure below shows two circles, with respective centers A and B . From the center of each circle, two tangents are drawn to the circumference of the other circle. The tangents from A intersect the circle centered at A at points C and D , and the tangents from B intersect the circle centered at B at the points E and F . Show that chords CD and EF are of equal length.



B10. Show that, for any five natural numbers a, b, c, d , and e , at least one of the ten pairs that can be formed from these five numbers is such that the difference of the squares of the numbers in this pair is divisible by 7.