

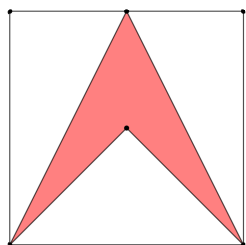
Mathematics and Statistics Awareness Month 2018

Middle School Level Problems

A1. In how many different ways can the number six be written as a sum of positive integers? Three ways to do this are: $3 + 1 + 1 + 1 = 6$ and $4 + 2 = 6$ and $1 + 2 + 3 = 6$. (Do not count reorderings as different. For example, $4 + 2$ is the same as $2 + 4$.)

How does the answer change if reorderings are counted as different?

A2. The arrow in the 2×2 square in the figure below is formed by drawing line segments connecting each bottom corner to the midpoint of the top side of the square and line segments connecting each bottom corner to the point in the center of the square. Find the area of the arrow.

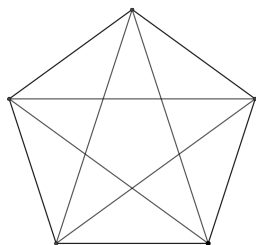


A3. For each of the 2-digit numbers: 41, 83, and 32, the first digit is greater in value than the second digit. How many 2-digit numbers have this property?

How many 3-digit numbers are there with the property that the first digit is greater in value than both the second digit and the third digit?

A4. Line segments connecting each pair of corners have been added to the pentagon in the following figure. How many triangles are there in this figure?

What would the answer be if we used a hexagon instead of a pentagon?



A5. The ratio of boys to girls at a school dance is 9:10. An extra 17 boys arrive and the ratio changes to 8:7. How many girls are there at the dance?

A6. Suppose we wrote all of the numbers from one to one hundred: 1, 2, 3, 4, ..., 99, 100.

- a) How many times would the number nine appear as a digit?
- b) If we added all the digits in these 100 numbers, what would we get as the sum?

A7. Mary wrote down the counting numbers from 1 to 20 in order and because each of the numbers from 10 to 20 contain two digits she noted that she had actually written down thirty-one digits in total. If her list went from 1 to 1000 how many digits would she have written down?

A8. The product of the digits in the number 126 is $1 \cdot 2 \cdot 6 = 12$.

- a) How many three digit numbers have this property that the product of the digits is 12?
- b) How many three digit numbers have the property that the product of the digits is 16?
- c) What is the first product that you cannot express as a product of the digits of a three digit number?

A9. Given a 2-digit number, multiply its digits together. If this process is continued (by multiplying the digits of the product), all 2-digit numbers will eventually become a single digit number. For example,

with 75: $7 \cdot 5 = 35$, $3 \cdot 5 = 15$, $1 \cdot 5 = 5$

with 68: $6 \cdot 8 = 48$, $4 \cdot 8 = 32$, $3 \cdot 2 = 6$

with 45: $4 \cdot 5 = 20$, $2 \cdot 0 = 0$

- a) How many 2-digit numbers have the property that this process results in a zero?
- b) Which numbers under 1000 have this property that this process results in a zero?

A10. There are one hundred and twenty 5-digit numbers with the property that each of the digits 1, 2, 3, 4, and 5 occurs exactly once in the number, how many of these numbers are divisible by 12?