Problems of the Month

University of Louisiana at Lafayette

February, 2017

Solutions must be submitted by 3/15/2017. They can be emailed or handed in to Calvin Berry (cberry@louisiana.edu) or Leonel Robert (lrobert@louisiana.edu).

1. This problem has two (related) parts.

a) Consider a set of five numbers $x_1 < x_2 < x_3 < x_4 < x_5$, with the property that there are only seven distinct values among the ten sums $x_i + x_j$ with $i \neq j$. Prove that x_1, x_2, x_3, x_4, x_5 form an arithmetic progression.

b) Consider a set of five numbers $0 < y_1 < y_2 < y_3 < y_4 < y_5$, with the property that there are only seven distinct values among the ten products $y_i y_j$ with $i \neq j$. Prove that y_1, y_2, y_3, y_4, y_5 form a geometric progression.

2. For integral n, prove that $\sin\sqrt{n+1} - \sin\sqrt{n}$ tends to zero as n tends to infinity.

3. In the figure below: The smaller circle is centered on the larger circle halfway between the endpoints of the chord BC of the larger circle. The smaller circle is tangent to the chord BC. Prove that the line through the upper endpoint of the chord BC which is tangent to the smaller circle is also tangent to the larger circle.