

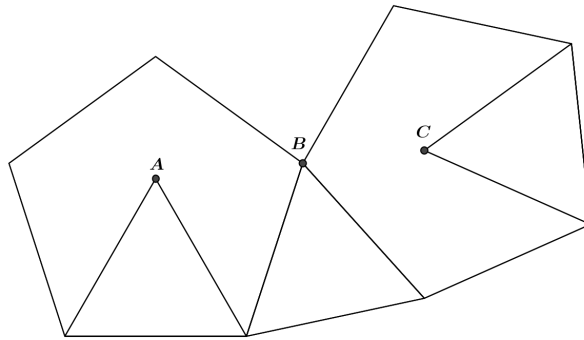
Problems of the Month

University of Louisiana at Lafayette

March, 2016

Solutions must be submitted by 04/15/2016. They can be emailed or handed in to Calvin Berry (cberry@louisiana.edu) or Leonel Robert (lrobert@louisiana.edu).

1. The triangles and pentagons in the figure are all regular with sides of one unit of length. Show that the points A , B , and C are on a line and find the distance from A to C .



2. Find the remainder in the division of 3^{2012} by 11.
3. Find the smallest number a such that $f(x) = \sin(x) - x + ax^3$ is an increasing function.
4. Let k be a natural number. Show that

$$\gcd\left(\binom{n}{k}, \binom{n+1}{k}, \dots, \binom{n+k}{k}\right) = 1$$

for any natural number $n \geq k$.

5. Let $z_1 = 1$ and $z_{n+1} = \frac{1}{2}(z_n + \frac{i}{z_n})$ for all $n \geq 1$ (where i is the imaginary unit). Show that $(z_n)_{n=1}^{\infty}$ converges and find its limit.