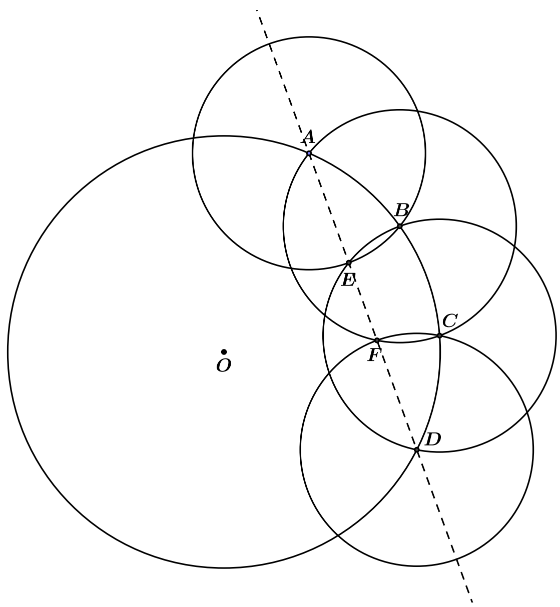
Problems of the Month

University of Louisiana at Lafayette

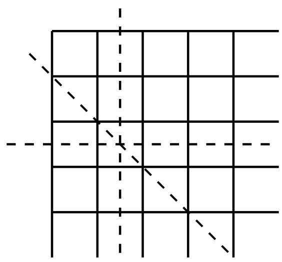
May, 2016

Solutions must be submitted by 06/20/2016. They can be emailed or handed in to Calvin Berry (cberry@louisiana.edu) or Leonel Robert (lrobert@louisiana.edu).

1. On a circle of center O choose four equidistant points A, B, C, D. Draw four circles with centers at these points and equal radius as shown in the picture. Show that the points A, E, F, D are on a line.



2. Show that the squares of an infinite array (as depicted below) can be filled with natural numbers $1, 2, 3, \ldots$ such that every row, column and diagonal of the array contains a permutation of all the natural numbers.



3. For $0 < p < 1$, the geometric distribution with parameter p has probability mass function

$$p(x) = P(X = x) = p(1-p)^x \mathbf{1}_{\{0,1,\dots\}}(x)$$

Let X denote a random variable with this geometric distribution, find $\Pr(X \geq x)$.

4. Let X and Y denote independent random variables each following the same geometric distribution with parameter p (see problem 3).
a. Derive the probability mass function for $W = \min(X, Y)$.
b. Find $P(\min(X, Y) = X) = P(X \leq Y)$.
c. Derive the probability mass function for $Z = X + Y$.
d. Given $z \in \{0, 1, \dots\}$, derive the conditional probability mass function for Y given $X + Y = z$, *i.e.*, find $P(Y = y | X + Y = z)$